Session 2: Review of Arithmetic of Fractions used in Drug Calculations

The arithmetic of fractions is very important groundwork which must be known before calculating the quantity of drugs to give to patients. Fractions are numbers like: \( \frac{1}{2}, \frac{3}{4}, \frac{17}{8} \) and so on.

**Fraction** = \( \frac{\text{Numerator}}{\text{Denominator}} = \frac{N}{D} \)

Suppose that N and D are both positive numbers. If N is less than D, the fraction is said to be a **proper fraction**. So, \( \frac{1}{2} \) and \( \frac{3}{4} \) are proper fractions since the numerator is less than the denominator. If N is greater than or equal to D, the fraction is said to be an **improper fraction**. So, \( \frac{11}{8}, \frac{7}{3}, \) and \( \frac{5}{5} \) are all improper fractions. Both proper and improper fractions are used for calculations of dosages of tablets, syrups, suspensions, injections and infusion rates.

**Exercise: 1**

Clarify each of the following as proper or improper fractions:

a) \( \frac{9}{17} \)  
b) \( \frac{5}{17} \)  
c) \( \frac{8}{8} \)

d) \( \frac{7}{8} \)  
e) \( \frac{110}{77} \)  
f) \( \frac{12}{7} \)

**Simplification**

Multiplying or dividing both numerator and denominator of a fraction by the same number produces a fraction having the same value, called an **equivalent fraction**.

A fraction is in its **simplest form** when there are no factors common to both numerator and denominator. For example \( \frac{5}{12} \) is in its simplest form but \( \frac{2}{4} \) is not since 2 is a factor common to both numerator and denominator. Its simplest form is the equivalent fraction \( \frac{1}{2} \).
To express a fraction in its simplest form we look for factors which are common to both the numerator and denominator. This is done by prime factorizing both of these. Dividing both the numerator and denominator by any common factors removes them but leaves an equivalent fraction. This is equivalent to cancelling any common factors. For example, to simplify \( \frac{4}{6} \) we prime factorize to produce:

\[
\frac{4}{6} = \frac{2 \times 2}{2 \times 3}
\]

Dividing both numerator and denominator by 2 leaves \( \frac{2}{3} \). This is equivalent to cancelling the common factor of 2.

**Worked Examples:**

1) Express \( \frac{24}{36} \) in its simplest form.

**Solution:** we seek factors common to both numerator and denominator. To do this we prime factorize 24 and 36.

\[
24 = 2 \times 2 \times 2 \times 3 \\
36 = 2 \times 2 \times 3 \times 3
\]

The factors \( 2 \times 2 \times 3 \) are common to both 24 and 36 and so these may be cancelled. Note that only common factors may be cancelled when simplifying a fraction. Hence:

\[
\frac{24}{36} = \frac{2 \times 2 \times 2 \times 3}{2 \times 2 \times 3 \times 3} = \frac{2}{3}
\]

In its simplest form \( \frac{24}{36} \) is \( \frac{2}{3} \). In effect we have divided 24 and 36 by 12, which is their common factor.

2) Express \( \frac{49}{21} \) in its simplest form.

**Solution:** Prime factorizing 49 and 21 gives:

\[
49 = 7 \times 7 \\
21 = 3 \times 7
\]
Their highest common factor is 7. Dividing 49 and 21 by 7 gives:

\[
\frac{49}{21} = \frac{7}{3}
\]

Hence the simplest form of \(\frac{49}{21} = \frac{7}{3}\)

**Exercises to work: 2.1**

1) Express the following in their simplest form:

a) \(\frac{24}{36}\)  b) \(\frac{49}{21}\)  c) \(\frac{18}{27}\)  d) \(\frac{15}{20}\)

e) \(\frac{2}{18}\)  f) \(\frac{30}{65}\)  g) \(\frac{12}{21}\)  h) \(\frac{12}{16}\)

2) Express \(\frac{3}{4}\) as an equivalent fraction having a denominator 28.

3) Express 4 as an equivalent fraction with a denominator of 5.

4) Express \(\frac{5}{12}\) as an equivalent fraction having a denominator of 36.

5) Simplify (cancel down) the following fractions:

a) \(\frac{21}{70}\)  b) \(\frac{28}{32}\)  c) \(\frac{9}{30}\)  d) \(\frac{15}{20}\)

e) \(\frac{75}{150}\)  f) \(\frac{125}{250}\)  g) \(\frac{175}{350}\)  h) \(\frac{17}{68}\)

i) \(\frac{30}{50}\)  j) \(\frac{80}{120}\)  k) \(\frac{100}{150}\)  l) \(\frac{60}{150}\)

m) \(\frac{100}{40}\)  n) \(\frac{1000}{80}\)  o) \(\frac{300}{120}\)  p) \(\frac{120}{2400}\)

q) \(\frac{0.4}{0.5}\)  r) \(\frac{0.35}{0.4}\)  s) \(\frac{100}{0.5}\)  t) \(\frac{0.04}{0.08}\)
Multiplication of fractions

The product of two or more fractions is found by multiplying their numerators to form a new numerator, and then multiplying their denominator to form a new denominator.

Worked Examples

1) \( \frac{4}{9} \times \frac{3}{8} \)

The numerators are multiplied: \( 4 \times 3 = 12 \)

The denominators are multiplied: \( 9 \times 8 = 72 \)

Hence: \( \frac{4}{9} \times \frac{3}{8} = \frac{12}{72} \)

This may now be expressed in its simplest form:

\[ \frac{12}{72} = \frac{12 \times 1}{12 \times 6} = \frac{1}{6} \]

Here the common factor 12 is cancelled out and the remainder \( \frac{1}{6} \) is the answer.

An alternative, but equivalent, method is to cancel any factors common to both numerator and denominator at the outset:

\[ \frac{4}{9} \times \frac{3}{8} = \frac{4 \times 3}{9 \times 8} \]

A factor of 4 is common to the 4 and the 8. Hence:

\[ \frac{4 \times 3}{9 \times 8} = \frac{1 \times 3}{9 \times 2} \]

A factor of 3 is common to the 3 and the 9. Hence:

\[ \frac{1 \times 3}{9 \times 2} = \frac{1 \times 1}{3 \times 2} = \frac{1}{6} \]
2) Find the simplification answer of:

\[
\frac{12}{25} \times \frac{2}{7} \times \frac{10}{9}
\]

**Solution:** we cancel factors common to both numerator and denominator.

A factor of 5 is common to 10 and 25. Cancelling this gives:

\[
\frac{12}{25} \times \frac{2}{7} \times \frac{10}{9} = \frac{12}{5} \times \frac{2}{7} \times \frac{2}{9}
\]

A factor of 3 is common to 12 and 9. Cancelling this gives:

\[
\frac{12}{5} \times \frac{2}{7} \times \frac{2}{9} = \frac{4}{5} \times \frac{2}{7} \times \frac{2}{3}
\]

There are no more common factors. Hence:

\[
\frac{12}{25} \times \frac{2}{7} \times \frac{10}{9} = \frac{4}{5} \times \frac{2}{7} \times \frac{2}{3} = \frac{16}{105}
\]

3) Find \(\frac{1}{2}\) of 10

**Solution:** \(\frac{1}{2}\) of 10 = \(\frac{1}{2} \times \frac{10}{1}\) (because we can write 10 as \(\frac{10}{1}\))

So: \(\frac{1}{2}\) of 10 = \(\frac{1}{2} \times \frac{10}{1} = \frac{10}{2} = \frac{5}{1} = 5\)

4) Find \(\frac{3}{4}\) of \(\frac{5}{9}\)

**Solution:** \(\frac{3}{4}\) of \(\frac{5}{9}\) is the same as: \(\frac{3}{4} \times \frac{5}{9}\)

Cancelling a factor of 3 from numerator and denominator gives \(\frac{1}{4} \times \frac{5}{3}\) that is \(\frac{5}{12}\)
Hence: \( \frac{3}{4} \) of \( \frac{5}{9} \) is \( \frac{5}{12} \)

**Exercises to work: 2.2**

1) Evaluate:
   
   a) \( \frac{2}{3} \times \frac{6}{7} \)
   
   b) \( \frac{8}{15} \times \frac{25}{32} \)
   
   c) \( \frac{1}{4} \times \frac{8}{9} \)
   
   d) \( \frac{16}{17} \times \frac{34}{48} \)
   
   e) \( 2 \times \frac{3}{5} \times \frac{5}{12} \)
   
   f) \( 2 \frac{1}{3} \times 1 \frac{1}{4} \)

2) Evaluate:
   
   a) \( \frac{2}{3} \) of \( \frac{3}{4} \)
   
   b) \( \frac{4}{7} \) of \( \frac{21}{30} \)
   
   c) \( \frac{9}{10} \) of 80
   
   d) \( \frac{6}{7} \) of 42

3) Is \( \frac{3}{4} \) of \( \frac{12}{15} \) the same as \( \frac{12}{15} \) of \( \frac{3}{4} \)?

4) Multiply the following fractions and simplify the answers:
   
   a) \( \frac{1}{2} \times \frac{2}{5} \)
   
   b) \( \frac{5}{6} \times \frac{8}{15} \)
   
   c) \( \frac{7}{8} \times \frac{8}{7} \)
   
   d) \( \frac{3}{4} \times \frac{20}{9} \)
   
   e) \( \frac{4}{7} \times \frac{5}{3} \)
   
   f) \( \frac{5}{9} \times \frac{21}{25} \)
   
   g) \( \frac{3}{5} \times \frac{3}{4} \)
   
   h) \( \frac{1}{8} \times \frac{1}{2} \)
   
   i) \( \frac{1}{10} \times \frac{15}{8} \)
5) Multiply and simplify where possible. Write each answer as a fraction, a mixed number or a whole number.

a) \( \frac{3}{4} \times 5 \)  
b) \( \frac{2}{7} \times 3 \)  
c) \( \frac{3}{5} \times 10 \)

d) \( \frac{3}{4} \times 6 \)  
e) \( \frac{2}{3} \times 5 \)  
f) \( \frac{5}{3} \times 4 \)

6) Multiply. Write each answer as a decimal number, or as a whole number, where this occurs.

a) \( \frac{6}{10} \times 2 \)  
b) \( \frac{18}{50} \times 5 \)  
c) \( \frac{35}{50} \times 4 \)

d) \( \frac{3}{10} \times 2 \)  
e) \( \frac{90}{50} \times 2 \)  
f) \( \frac{7}{20} \times 2 \)